

Практическое занятие № 7
Вычисление определенного интеграла

Цель: Повторение, обобщение и систематизация материала по данной теме.

Задания

1. Вычислить определенный интеграл методом непосредственного интегрирования

1. a) $\int_1^2 (3x^2 - 2x) dx$

b) $\int_{\pi}^{2\pi} \frac{\cos x}{6} dx$

2. a) $\int_{-2}^1 (x^2 - x) dx$

b) $\int_{-\pi/6}^{\pi/6} \frac{6dx}{\cos^2 2x}$

3. a) $\int_{-\pi/2}^{\pi/2} 3 \cos x dx$

b) $\int_0^2 (1 - \frac{x}{2})^4 dx$

4. a) $\int_0^{\pi/4} \frac{3}{\cos^2 x} dx$

b) $\int_{-1}^4 (1 + \frac{x}{2})^8 dx$

5. a) $\int_1^2 (4x^3 + 2x) dx$

b) $\int_0^{\pi} \frac{3dx}{\cos^2(\frac{x}{2} - \frac{\pi}{3})}$

6. a) $\int_0^{\pi/4} (4 \cos 2x) dx$

b) $\int_1^5 \sqrt{9x-1} dx$

11. a) $\int_0^{\pi/2} \frac{3}{\cos^2 \frac{1}{2}x} dx$

b) $\int_2^3 (1-x)^4 dx$

12. a) $\int_0^{\frac{3\pi}{2}} \cos \frac{1}{3}x dx$

b) $\int_{-1}^4 (1 + \frac{x}{2})^8 dx$

13. a) $\int_2^3 (3x^2 - 2x) dx$

b) $\int_{-\pi/2}^{\pi/2} 3 \sin x dx$

14. a) $\int_0^{\pi} (3 \sin \frac{1}{2}x) dx$

b) $\int_1^0 (1-2x)^4 dx$

15. a) $\int_0^{\pi/4} (36 \cos 2x) dx$

b) $\int_{-2}^3 \frac{2dx}{(3-x^2)}$

16. a) $\int_0^{\pi/6} \cos x dx$

b) $\int_2^3 (1-2x)^4 dx$

21. a) $\int_1^2 (4x^3 - 3x^2) dx$

b) $\int_0^4 \frac{dx}{\sqrt{x+3}}$

22. a) $\int_0^{\pi/9} (2 \cos 3x) dx$

b) $\int_0^2 (1 - \frac{x}{2})^4 dx$

23. a) $\int_0^{\pi/12} (108 \sin 6x) dx$

b) $\int_{-1}^1 (7-5x) dx$

24. a) $\int_0^{\pi/8} \frac{4}{\cos^2 2x} dx$

b) $\int_{-2}^1 (4x^3 + 6x) dx$

25. a) $\int_1^2 (5x^4 - 6x^2) dx$

b) $\int_1^9 \sqrt{8x-5} dx$

26. a) $\int_{\pi/6}^{\pi/2} \frac{5}{\sin^2 x} dx$

b) $\int_{-2}^0 (x^5 - 3x^2)$

$$7. \quad a) \int_{\frac{\pi}{2}}^{\pi} (6 \sin 2x) dx$$

$$b) \int_1^2 (3-2x)^4 dx$$

$$8. \quad a) \int_0^{\frac{\pi}{3}} \frac{5}{\cos^2 x} dx$$

$$b) \int_{-3}^1 \frac{dx}{5-3x}$$

$$9. \quad a) \int_0^2 (x^3 - x^2 + 3) dx$$

$$b) \int_1^2 e^{2x+3} dx$$

$$10. \quad a) \int_1^2 (x^4 - x^3 + 2) dx$$

$$b) \int_0^1 5^{4-3x} dx$$

$$17. \quad a) \int_0^{\frac{\pi}{8}} \frac{36}{\cos^2 2x} dx$$

$$b) \int_2^3 (3-x^2) dx$$

$$18. \quad a) \int_0^{\frac{\pi}{2}} (2 \sin x) dx$$

$$b) \int_0^4 (x^2 + 1) dx$$

$$19. \quad a) \int_0^{\frac{\pi}{3}} \sin x dx$$

$$b) \int_2^3 (7-2x)^4 dx$$

$$20. \quad a) \int_0^2 (x^3 - x) dx$$

$$b) \int_0^{\frac{\pi}{3}} 3 \sin 3x dx$$

$$27. \quad a) \int_0^{\frac{\pi}{3}} (2 \sin x) dx$$

$$b) \int_1^2 \left(\frac{x^2}{2} - \frac{2}{x^3} \right) dx$$

$$28. \quad a) \int_1^3 (3x^2 + 4x^3) dx$$

$$b) \int_{\frac{\pi}{4}}^{\frac{\pi}{8}} \frac{\cos 4x}{2} dx$$

$$29. \quad a) \int_2^3 (2x^3 - 2x) dx$$

$$b) \int_0^{\frac{\pi}{6}} \frac{4dx}{\cos^2 2x}$$

$$30. \quad a) \int_{\frac{\pi}{18}}^{\frac{\pi}{6}} (2 \sin 3x) dx$$

$$b) \int_0^1 \frac{dx}{x^2 + 4}$$

2. Вычислить определенный интеграл методом замены переменной

$$1. \quad \int_0^{\sqrt{3}} x^3 \sqrt{1+x^2} dx$$

$$11. \quad \int_0^1 x^3 \sqrt{4+5x^4} dx$$

$$21. \quad \int_1^2 \frac{e^{1/x}}{x^2} dx$$

$$2. \quad \int_0^{1/2} \frac{xdx}{\sqrt{1-x^2}}$$

$$12. \quad \int_1^e \frac{1+\ln x}{x} dx$$

$$22. \quad \int_1^{\sqrt{2}} \frac{xdx}{\sqrt{4-x^2}}$$

$$3. \quad \int_1^e \frac{\ln^2 x}{x} dx$$

$$13. \quad \int_0^1 \frac{dx}{\sqrt[3]{2-4x}}$$

$$23. \quad \int_0^1 \frac{x^3 dx}{\sqrt{16x^4+1}}$$

$$4. \quad \int_{-\infty}^0 \frac{xdx}{\sqrt{(x^2+4)^3}}$$

$$14. \quad \int_{1/3}^1 \frac{\ln(3x-1)}{3x-1} dx$$

$$24. \quad \int_0^1 \frac{x^2 dx}{\sqrt[3]{(x^3+8)^4}}$$

$$5. \quad \int_0^1 \frac{xdx}{\sqrt[4]{(16+x^2)^5}}$$

$$15. \quad \int_0^{2/3} \frac{\sqrt[3]{\ln(2-3x)}}{2-3x} dx$$

$$25. \quad \int_0^{\pi/6} \frac{\cos 3x}{\sqrt[6]{(1-\sin 3x)^5}} dx$$

$$6. \quad \int_{3/4}^1 \frac{dx}{\sqrt[5]{3-4x}}$$

$$16. \quad \int_{\pi/2}^{\pi} \frac{\sin x dx}{\sqrt[7]{\cos^2 x}}$$

$$26. \quad \int_{-3/4}^0 \frac{dx}{\sqrt{4x+3}}$$

$$7. \quad \int_0^1 \frac{x^4 dx}{\sqrt[3]{1-x^5}}$$

$$17. \quad \int_{1/2}^1 \frac{dx}{\sqrt[9]{1-2x}}$$

$$27. \quad \int_1^5 \frac{x^2 dx}{\sqrt{31(x^3-1)}}$$

$$8. \quad \int_0^4 \frac{10xdx}{\sqrt[4]{(16-x^2)^3}}$$

$$18. \quad \int_0^{1/4} \frac{dx}{\sqrt[3]{1-4x}}$$

$$28. \quad \int_0^{1/2} \frac{dx}{(2x-1)^2}$$

$$9. \quad \int_0^2 \frac{x^3 dx}{\sqrt{(x^4+4)}}$$

$$19. \quad \int_1^e \frac{1+\ln x}{x} dx$$

$$29. \quad \int_0^1 x^3 \sqrt{4+5x^4} dx$$

$$10. \int_0^{\frac{1}{2}} \frac{x dx}{\sqrt{1-x^2}} \quad 20. \int_1^{\sqrt{e}} \frac{dx}{x\sqrt{1-\ln^2 x}} \quad 30. \int_{\pi/2}^{\pi} \frac{\sin x dx}{\sqrt{\cos^2 x}}$$

3. Вычислить определенный интеграл

$$\begin{array}{lll} 1. \int_0^1 \frac{3x^4 + 3x^2 + 1}{x^2 + 1} dx & 11. \int_2^3 \frac{dx}{(x-1)^2(x+1)} & 21. \int_4^5 \frac{dx}{x^2(x-1)} \\ 2. \int_2^3 \frac{2x^4 - 5x^2 + 3}{x^2 - 1} dx & 12. \int_3^5 \frac{(x^2 + 2)dx}{(x^2 + 1)(x-1)} & 22. \int_0^2 \frac{dx}{(x+1)(x^2 + 4)} \\ 3. \int_2^3 \frac{x+2}{x^2(x-1)} dx & 13. \int_0^1 \frac{x^4 + 3x^3 - 1}{(x+1)^2} dx & 23. \int_7^9 \frac{x^2 - x + 2}{x^4 - 5x^2 + 4} dx \\ 4. \int_2^3 \frac{dx}{x^2(x-1)} & 14. \int_0^1 \frac{x dx}{x^2 + 3x + 2} & 24. \int_4^6 \frac{x dx}{x^3 - 6x^2 + 16 - 6} \\ 5. \int_{-1}^1 \frac{y^5 dy}{y+2} & 15. \int_8^{10} \frac{(x^2 + 3)dx}{x^3 - x^2 - 6x} & 25. \int_1^2 \frac{dx}{x^3 + 1} \\ 6. \int_2^3 \frac{3x^2 + 2x - 3}{x^3 - x} dx & 16. \int_1^{\sqrt{3}} \frac{dx}{x^4 + x^2} & 26. \int_1^{\sqrt{3}} \frac{x^5 + 1}{x^6 + x^4} dx \\ 7. \int_{1/3}^{1/2} \frac{x dx}{(x-1)^3} & 17. \int_2^3 \frac{x^7 dx}{1-x^4} & 27. \int_2^3 \frac{x^3 + x^2 + 2}{x(x^2 - 1)^2} dx \\ 8. \int_4^5 \frac{dx}{(x-1)(x+2)} & 18. \int_2^3 \frac{dx}{x^4 - 1} & 28. \int_3^5 \frac{x^3 - 2x^2 + 4}{x^3(x-2)^2} dx \\ 9. \int_3^4 \frac{dx}{(x+1)(x-2)} & 19. \int_{-1}^0 \frac{x dx}{x^3 - 1} & 29. \int_0^{1/\sqrt{3}} \frac{x^2 dx}{x^4 - 1} \\ 10. \int_0^1 \frac{(2x+3)dx}{(x-3)^3} & 20. \int_0^{\sqrt{3}/3} \frac{2x^2 + 4}{x^3 - x^2 + x + 1} dx & 30. \int_{-1}^0 \frac{x^5 - 2x^2 + 3}{(x-2)^2} dx \end{array}$$

Задание 4

$$\begin{array}{lll} 1.1 \quad a) \int_0^{\sqrt{3}} x^3 \sqrt{1+x^2} dx & a) \int_{-1}^2 \frac{6x^2 dx}{(x^3 - 5)^2} & a) \int_{2\sqrt{2}}^4 3x \sqrt{x^2 - 7} dx \\ b) \int_0^2 (2-x)^2 dx & b) \int_0^{\frac{\pi}{4}} \sqrt{2} \cos x dx & b) \int_0^{\frac{\pi}{2}} 2 \sin x dx \\ 1.2 \quad a) \int_1^3 \frac{dx}{\sqrt[3]{(3-x)^3}} & a) \int_0^1 \frac{x^3 dx}{\sqrt{16x^4 + 1}} & a) \int_{1/2}^1 \frac{dx}{\sqrt[9]{1-2x}} \\ b) \int_0^{\frac{\pi}{2}} \frac{\cos x dx}{(3 - \sin x)^2} & b) \int_0^{\sqrt{3}} x \sqrt{25 - 3x^2} dx & b) \int_0^{\frac{\pi}{2}} \frac{\cos x dx}{(8 - 7 \sin x)^2} \end{array}$$

- 1.3 a) $\int_{-1/3}^0 \frac{dx}{\sqrt[3]{1+3x}}$
b) $\int_1^2 \frac{(2x^2+1)dx}{x}$
- 1.4 a) $\int_{3/4}^1 \frac{dx}{\sqrt[3]{3-4x}}$
b) $\int_1^2 \frac{xdx}{(2x^2+4)^4}$
- 1.5 a) $\int_0^1 \frac{x^4 dx}{\sqrt[3]{1-x^5}}$
b) $\int_0^{\pi/2} \frac{\cos x dx}{(5-\sin x)^2}$
- 1.6 a) $\int_{3/4}^{4/3} \frac{dx}{x^2+1}$
b) $\int_0^{\sqrt{3}} x^5 \sqrt{1+x^2} dx$
- 1.7 a) $\int_0^{-3} \frac{dx}{\sqrt{25+3x}}$
b) $\int_0^{\pi/6} \frac{\cos 3x}{\sqrt[6]{(1-\sin 3x)^5}} dx$
- 1.8 a) $\int_0^2 \frac{x^3 dx}{\sqrt{x^4+4}}$
b) $\int_0^{\pi/2} 3 \cos 2x dx$
- 1.9 a) $\int_1^e \frac{3 \ln^2 x}{x} dx$
b) $\int_0^1 \frac{x^2 dx}{\sqrt[5]{8-7x^3}}$
- 1.10 a) $\int_0^1 \frac{dx}{\sqrt[3]{2-4x}}$
b) $\int_0^{\pi/2} \frac{\cos x dx}{(8-\sin x)^2}$
- 1.13 a) $\int_0^1 x^3 \sqrt{4+5x^4} dx$
b) $\int_1^2 \frac{(x-1)dx}{x^3}$
- 1.14 a) $\int_0^3 6x^3(3x^4-1)^2 dx$
b) $\int_0^1 \frac{x^2 dx}{\sqrt[3]{8-7x^3}}$
- 1.15 a) $\int_0^1 (2x^3+1)^4 x^2 dx$
b) $\int_0^2 \frac{x^2 dx}{\sqrt[3]{9+2x^3}}$
- 1.16 a) $\int_0^{1/2} \frac{xdx}{\sqrt{1-x^2}}$
b) $\int_4^9 \frac{\sqrt{x}}{\sqrt{x-1}} dx$
- 1.17 a) $\int_0^1 (e^x-1)^4 e^x dx$
b) $\int_0^{1/4} \frac{dx}{\sqrt[3]{1-4x}}$
- 1.18 a) $\int_{-3/4}^0 \frac{dx}{\sqrt{4x+3}}$
b) $\int_0^4 \frac{10xdx}{\sqrt[4]{(16-x^2)^3}}$
- 1.19 a) $\int_0^1 (4x^3+1)^5 x^2 dx$
b) $\int_0^1 x^2 e^{x^3+1} dx$
- 1.20 a) $\int_0^{\pi/6} \frac{\cos 3x}{\sqrt[6]{(1-\sin 3x)^5}} dx$
b) $\int_1^{\sqrt{3}} \frac{32xdx}{(x^2+1)^5}$
- 1.23 a) $\int_1^{\sqrt{2}} \frac{xdx}{\sqrt{4-x^2}}$
b) $\int_1^{e^{e^3}} \frac{dx}{x\sqrt{1+\ln x}}$
- 1.24 a) $\int_{-1}^0 \frac{dx}{4x^2-9}$
b) $\int_1^3 \sqrt{(2x+1)^3} dx$
- 1.25 a) $\int_0^{1/4} \frac{dx}{\sqrt[3]{1-4x}}$
b) $\int_4^5 (4-x)^3 dx$
- 1.26 a) $\int_0^{1/2} \frac{dx}{(2x-1)^2}$
b) $\int_1^e \frac{5 \ln^2 x}{x} dx$
- 1.27 a) $\int_0^1 \frac{dx}{\sqrt{4-3x}}$
b) $\int_0^1 (x^3-4) dx$
- 1.28 a) $\int_3^8 \sqrt{x+1} dx$
b) $\int_0^2 \frac{xdx}{(x^2-1)^3}$
- 1.29 a) $\int_0^4 \frac{10xdx}{\sqrt[4]{(16-x^2)^3}}$
b) $\int_{\frac{2\pi}{9}}^{\frac{\pi}{3}} \sin\left(3x-\frac{\pi}{2}\right) dx$
- 1.30 a) $\int_1^5 \frac{x^2 dx}{\sqrt{31(x^3-1)}}$
b) $\int_0^1 e^{x^2} x dx$

2.1	$\int_2^3 x \ln(x-1) dx$	$\int_1^{e^2} \sqrt{x} \ln x dx$	$\int_2^3 (3-x)e^x dx$
2.2	$\int_{-2}^0 x^2 e^{-x/2} dx$	$\int_0^1 \operatorname{arctg} \sqrt{x} dx$	$\int_{1/2}^1 \arcsin(1-x) dx$
2.3	$\int_0^{\pi/2} x \cos x dx$	$\int_0^{\pi} (x+2) \cos \frac{x}{2} dx$	$\int_1^{\sqrt{3}} \operatorname{arctg} \frac{1}{x} dx$
2.4	$\int_0^{\pi/2} x^2 \sin x dx$	$\int_0^{\pi/8} x^2 \sin 4x dx$	$\int_{-1}^0 x \ln(1-x) dx$
2.5	$\int_{-1/2}^{1/2} \arccos 2x dx$	$\int_1^2 x^2 \ln x dx$	$\int_0^{\pi/6} e^{\sin x} \cos x dx$
2.6	$\int_1^2 (x-1) \ln x dx$	$\int_1^e (x+1) \ln x dx$	$\int_{-1}^0 (2x+3)e^{-x} dx$
2.7	$\int_{-1/2}^0 x e^{-2x} dx$	$\int_{3/2}^2 \operatorname{arctg}(2x-3) dx$	$\int_{-1}^0 (x+1)e^{-2x} dx$
2.8	$\int_1^2 x e^x dx$	$\int_0^{\pi/2} (x+3) \sin x dx$	$\int_{-1/2}^{1/2} \arccos 2x dx$
2.9	$\int_{-1/3}^{-2/3} \frac{x}{e^{3x}} dx$	$\int_0^{\ln 2} x e^{-x} dx$	$\int_0^1 x \operatorname{arctg} x dx$
2.10	$\int_1^e \frac{\ln^2 x}{x^2} dx$	$\int_{-3}^0 (x-2)e^{-x/3} dx$	$\int_1^2 \ln(3x+2) dx$

Контрольные вопросы

1. Что называют определенным интегралом функции $f(x)$?
2. В чем состоит геометрический смысл определенного интеграла?
3. Сформулируйте необходимое условие интегрируемости функции $f(x)$ на отрезке $[a, b]$.
4. Сформулируйте необходимые и достаточные условия интегрируемости функции $f(x)$ на отрезке $[a, b]$.
5. Запишите свойства определенного интеграла.
6. Запишите формулу Ньютона-Лейбница.

Расскажите об основных методах интегрирования определенного интеграла.